Determination of Spin and Parity of Hyperons Using Polarized Proton Target*

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(Received 10 February 1964)

Relations are derived to determine the spin and relative intrinsic parity of a fermion, such as the cascade particle, produced in association with a spinless boson by the reaction of a second spinless particle with a polarized proton target. Inequalities are derived for general spin parity assignments between P, the polarization of the final fermions, when produced from an unpolarized target; P', the dependence of the rate of the reaction upon target polarization; and D, the correlation between initial and final polarization. Use is made of Bohr's theorem of R invariance.

THE development of polarized targets makes possible the unambiguous determination of both the spin and parity of hyperons that decay via parity nonconserving interactions. The method used is an extension of the ideas put forward by Lee and Yang,[1] and by Teutsch, Okubo, and Sudarshan.[2]

We consider reactions of the type

$$a+p\rightarrow b+Y \quad (1a)$$

followed by

$$Y\rightarrow B+c, \quad (1b)$$

where a, b, and c are spinless bosons, p and B are spin-$\frac{1}{2}$ baryons, and Y is a hyperon of unknown spin. Examples of this type of reaction are

$$\pi^+ + p\rightarrow K^+ + \Sigma^+, \quad (2a)$$

$$\Sigma^+ \rightarrow p+\pi^0, \quad (2b)$$

and

$$K^- + p\rightarrow K^+ + \Xi^- \quad (3a)$$

$$\Xi^- \rightarrow \Lambda^0 + \pi^-. \quad (3b)$$

We choose the axis of spin quantization in the direction $\hat{n}=P_a\times P_b/|P_a\times P_b|$ normal to the plane of reaction (1a). The particle Y has spin J, as yet to be determined. Let the relative population of the level with spin-projection m be I_m. I_m is a function of the energy and angle of reaction (1a), as well as the spin direction of the target proton. Let I_m^0 be the value of I_m at a particular energy and angle (or averaged over a suitable range) when the target proton is unpolarized. The I_m and the I_m^0, obey the following relations:

$$I_m \geqslant 0, \quad (4)$$

$$\sum_{m=-J}^{m=+J} I_m = 1. \quad (5)$$

The two measurements that are required to make the spin-parity determination are (1) P, the polarization of the Y in (1a) when the target proton is unpolarized; and (2) P', the dependence of the rate of (1a) on the polarization of the proton, without observing any of the final spin states. Both these quantities are to be averaged over the same energy and angular range. More specifically

$$P'=\frac{\text{rate}(m_p=+\frac{1}{2})-\text{rate}(m_p=-\frac{1}{2})}{\text{rate}(m_p=+\frac{1}{2})+\text{rate}(m_p=-\frac{1}{2})}, \quad (6)$$

where m_p is the projection of the proton spin along the $\hat{n}$ direction. P' is measured using a polarized target.

$$P=(1/J)\sum mI_m. \quad (7)$$

Lee and Yang[2] have shown that P can be determined using reaction (1b). The expectation value of the cosine of the angle between the momentum of particle B, in the rest frame of the Y, and the direction $\hat{n}$, is given by

$$\langle \hat{n}\cdot\hat{B}\rangle=\frac{\alpha}{2J(J+1)}\sum mI_m=\frac{\alpha}{2(J+1)}P. \quad (8)$$

α is the usual parameter that shows the amount of parity mixing in reaction (1b). The longitudinal polarization of B, when Y is unpolarized (or averaged over all polarizations of Y), is equal to α, independent of J. Therefore, P can be measured, when $\alpha\neq 0$, according to Eq. (8)—for any hypothesized J—in the usual type of bubble chamber experiment.

Let the relative intrinsic parities of the particles in (1a) be π_a, π_b, π_p, π_Y, and let

$$\Pi=\pi_Y\pi_b/\pi_a\pi_p. \quad (9)$$

We will now derive the following inequality

$$|(2JP-\omega P')|\leqslant 2J-1, \quad (10)$$

where $\omega=\Pi(-1)^{J-1/2}$

$$\omega=+1 \quad \text{for} \quad J^\Pi=\tfrac{1}{2}^+,\ \tfrac{3}{2}^-,\ \tfrac{5}{2}^+,\ \text{etc.},$$

$$\omega=-1 \quad \text{for} \quad J^\Pi=\tfrac{1}{2}^-,\ \tfrac{3}{2}^+,\ \tfrac{5}{2}^-,\ \text{etc.}$$

The proof of the inequality (10) is based on a theorem by Bohr.[3] Let all the particles in reaction (1a) be de-

* Work done under the auspices of the U. S. Atomic Energy Commission.
[1] T. D. Lee and C. N. Yang, Phys. Rev. **109**, 1755 (1958).
[2] W. B. Teutsch, S. Okubo, and E. C. G. Sudarshan, Phys. Rev. **114**, 1148 (1959).
[3] A. Bohr, Nucl. Phys. **10**, 486 (1959).

scribed in terms of plane waves with spin quantized along the $\hat{n}$ direction. These wave functions are eigenstates of the operator R, combined inversion and 180° rotation about the n axis, which commutes with the strong-interaction Hamiltonian. This leads to the selection rule

$$\pi_a \pi_p e^{i\pi m_p} = \pi_b \pi_Y e^{i\pi m},$$

where $m_p = \pm\frac{1}{2}$ is the spin-projection eigenvalue of the target proton. This means that the population I_m of the Y state with spin projection m is contributed entirely by one of the proton spin states and not the other. As a consequence of this, we may write

$$\frac{\text{rate}(m_p \to m)}{\text{rate}(+\frac{1}{2} \to \text{all } m) + \text{rate}(-\frac{1}{2} \to \text{all } m)} = I_m^0 \quad \text{if} \quad \Pi(-1)^{m-m_p} = +1$$
$$= 0 \quad \text{if} \quad \Pi(-1)^{m-m_p} = -1, \quad (11)$$

from which we confirm that $I_m = I_m^0$ for an unpolarized target. From this and Eq. (6) we deduce

$$P' = \sum_{-J}^{J} \Pi(-1)^{m-\frac{1}{2}} I_m^0. \quad (12)$$

For example, in the case $J=\frac{3}{2}$, $\Pi=+1$, we have

$$P' = -(I_{3/2}^0 - I_{-3/2}^0) + (I_{1/2}^0 - I_{-1/2}^0),$$

whereas from Eq. (7),

$$P = (I_{3/2}^0 - I_{-3/2}^0) + \tfrac{1}{3}(I_{1/2}^0 - I_{-1/2}^0).$$

It is now simple to form

$$2JP - \omega P' = (2J-1)(I_J^0 - I_{-J}^0 + I_{J-1}^0 - I_{-J+1}^0)$$
$$+ (2J-5)(I_{J-2}^0 - I_{-J+2}^0 + I_{J-3}^0 - I_{-J+3}^0)$$
$$+ (2J-9)(\qquad) + \cdots. \quad (13)$$

The right-hand side of Eq. (13) cannot have magnitude greater than $(2J-1)$, because of the constraints (4) and (5) on the I_m. This completes the proof.

When $J=\frac{1}{2}$, $2J-1=0$, and inequality (10) reduces to the equation

$$P' = \Pi P,$$

a result that was first derived by Bilenky.[4]

When $J=\frac{3}{2}$, we have

$$|P'+3P| \leqslant 2 \quad \text{for} \quad \Pi=+1,$$
$$|P'-3P| \leqslant 2 \quad \text{for} \quad \Pi=-1.$$

The results for spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ are illustrated together in Fig. 1. The ordinate is P', and the abscissa is $\langle\hat{n}\cdot\hat{B}\rangle/\alpha = P/(2J+2)$, which is the experimentally measured quantity.

Table I gives the inequalities applying to some of the low spin-parity assignments.

These inequalities are stronger than those derived previously[1,2] and include them as a limiting case (either for $J\to\infty$ or for $P'=\pm1$). Further limits can be deduced by measuring the polarization of Y particles produced from polarized targets. If we define

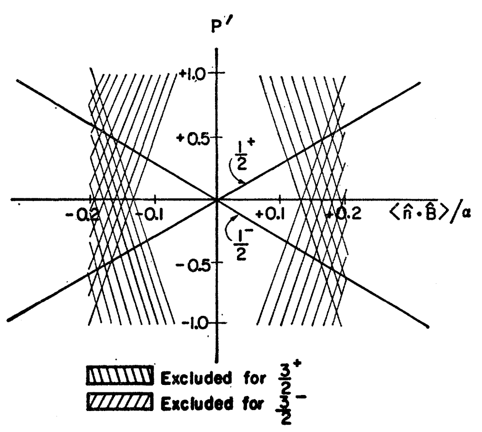

FIG. 1. Relations between P and P' for spin $\frac{1}{2}$ and $\frac{3}{2}$. The abscissa is the experimentally measured $\langle\hat{n}\cdot\hat{B}\rangle/\alpha = P/(2J+2)$.

TABLE I. Relations between P and P'.

	$\Pi=+1$	$\Pi=-1$
$J=\frac{1}{2}$	$P'=P$	$P'=-P$
$J=\frac{3}{2}$	$\lvert P'+3P\rvert \leqslant 2$	$\lvert P'-3P\rvert \leqslant 2$
$J=\frac{5}{2}$	$\lvert P'-5P\rvert \leqslant 4$	$\lvert P'+5P\rvert \leqslant 4$
$J=\frac{7}{2}$	$\lvert P'+7P\rvert \leqslant 6$	$\lvert P'-7P\rvert \leqslant 6$
	etc.	

$$D = \frac{(\text{rate}\times P)(m_p=+\frac{1}{2}) - (\text{rate}\times P)(m_p=-\frac{1}{2})}{\text{rate}(m_p=+\frac{1}{2}) + \text{rate}(m_p=-\frac{1}{2})}, \quad (14)$$

where P is defined in Eq. (7), and can be measured according to Eq. (8). Arguments similar to those which led to Eq. (12) show that

$$D = (1/J) \sum_{m=-J}^{J} \Pi(-1)^{m-\frac{1}{2}} m I_m^0. \quad (15)$$

When $J=\frac{1}{2}$, $D=\Pi$, i.e., $D=\pm1$ depending on the relative parity. Note that this is true even if P and P' are zero.

When $J=\frac{3}{2}$, we can solve Eqs. (5), (7), (12), and (15) simultaneously for the four I_m^0, then apply (4) to each of them to obtain the inequalities, when $\Pi=+1$,

$$|3P-P'| \leqslant 1-3D, \quad (16)$$
$$|P+P'| \leqslant 1+D, \quad (17)$$

[4] S. M. Bilenky, Nuovo Cimento 10, 1049 (1958).

from which one can derive the weaker inequalities, $|P| \leqslant \frac{1}{2}(1-D)$, $-1 \leqslant D \leqslant +\frac{1}{3}$, and Eq. (10). When $\Pi = -1$, the appropriate relations are obtained from (16) and (17) with the signs of P' and D reversed.

The generalizations of (16) and (17) to arbitrary J and Π are

$$|JP+(J-1)\omega P'| \leqslant J-1+J\omega D \tag{18}$$

and

$$|P-\omega P'| \leqslant 1-\omega D. \tag{19}$$

From (18) and (19) one can derive weaker inequalities. By adding (19) to twice (18) one derives, for $J \neq \frac{1}{2}$,

$$|P+\omega P'| \leqslant 1+\omega D. \tag{20}$$

This means that, independent of ω, for all spin-parity assignments except $J=\frac{1}{2}$,

$$|P \pm P'| \leqslant 1 \pm D. \tag{21}$$

Since (21) is always true, no new information is yielded by use of it. The stronger inequality is obtained by the choice of sign conforming to relation (19).

Other inequalities implied by (18) and (19) are

$$\omega D + (J-1)/J \leqslant 0 \tag{22}$$

since the right-hand side of (18) must be positive.

$$(2J-1)|P| \leqslant 2(J-1)+\omega D \tag{23}$$

is obtained by adding (18) to $(J-1)$ times (19).

Relation (10) can be derived by adding (19) to J times (18).

TABLE II. Relations involving P, P', and D.

	$\Pi=+1$	$\Pi=-1$
$J=\frac{1}{2}$	$D=+1$	$D=-1$
$J=\frac{3}{2}$	$\lvert 3P-P'\rvert \leqslant 1-3D$ $-1 \leqslant D \leqslant +\frac{1}{3}$ $\lvert P\rvert \leqslant \frac{1}{2}(1-D)$	$\lvert 3P+P'\rvert \leqslant 1+3D$ $-\frac{1}{3} \leqslant D \leqslant +1$ $\lvert P\rvert \leqslant \frac{1}{2}(1+D)$
$J=\frac{5}{2}$	$\lvert 5P+3P'\rvert \leqslant 3+5D$ $-\frac{3}{5} \leqslant D \leqslant +1$ $\lvert P\rvert \leqslant \frac{1}{4}(3+D)$	$\lvert 5P-3P'\rvert < 3-5D$ $-1 \leqslant D \leqslant +\frac{3}{5}$ $\lvert P\rvert \leqslant \frac{1}{4}(3-D)$
$J=\frac{7}{2}$	$\lvert 7P-5P'\rvert \leqslant 5-7D$ $-1 \leqslant D \leqslant +(5/7)$ $\lvert P\rvert \leqslant \frac{1}{6}(5-D)$	$\lvert 7P+5P'\rvert \leqslant 5+7D$ $-(5/7) \leqslant D \leqslant +1$ $\lvert P\rvert \leqslant \frac{1}{6}(5+D)$

Inequalities involving D are presented for the lowest J values in Table II.

Recently, Ademollo and Gatto[5] and Peshkin[6] have treated the problem of spin-parity determination in reactions similar to (1a), (1b), and making use of R invariance. These treatments do not consider the case of polarized targets. Gaillard[7] has treated the application of polarized targets to the determination of spins and parities of particle resonances.

The author wishes to thank Professor O. Chamberlain for suggesting this problem, and Dr. H. P. Stapp for reading the manuscript.

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[6] M. Peshkin, Phys. Rev. **133**, B428 (1964).
[7] M. K. Gaillard, CERN internal report 7278/TH. 375 (unpublished).